1 OccBin Filter

- The original OccBin file solve_oneconstraint.m solves the model given a specific set of shocks
- This file can be modified to solve for the shocks that would deliver a specific set of observables
- In OccBin,
  - given a $T_{\max}$ representing the last period in which the constraint binds
  - and hence a guess for all the periods in which the constraint is expected to bind
  - the code *mkdatap_anticipated.m* uses the policy functions obtained from dynare (A, B, C, ϵ and $A^*, B^*, C^*, D^*, \epsilon^*$) in order to obtain the sequence of matrices

    $$\{P_t\}_{t=1}^{T_{\max}}, \{R_t\}_{t=1}^{T_{\max}}, \{Q_t\}_{t=1}^{T_{shock}}$$

    for the time varying representation

    $$X_t = P_t X_{t-1} + R_t + Q_t \varepsilon_t \quad \text{for } t \in T_{shock}$$

    $$X_t = P_t X_{t-1} + R_t \text{ for } t > T_{shock}$$

    where X is $n \times 1, \varepsilon$ is $n_\varepsilon \times 1$ and P is $n \times n$,R is $n \times 1$ and Q is $n \times n_\varepsilon$

- Assume that we have a set of observables Y_t

  $$Y_t = HX_t \quad \text{where } H \text{ is } m \times n$$

- Then, we can rewrite

  $$\overset{m\times 1}{Y}_t = \overset{m\times n}{H} * \overset{n\times n}{P}_t * \overset{n\times 1}{X}_{t-1} + \overset{m\times n}{H} \overset{n\times 1}{R}_t + \overset{m\times n}{H} * \overset{n\times n_\varepsilon}{Q}_t * \overset{n_\varepsilon\times 1}{\varepsilon}_t$$

- In addition we need to select a subset of shocks that is equal to the number of observables, in order to invert them out, that is we have to substitute $\overset{n\times n_\varepsilon}{Q}_t$with $\overset{n\times m}{\tilde{Q}}_t$which selects only the columns associated with the shocks we need so that

  $$\overset{m\times 1}{Y}_t = \overset{m\times n}{H} * \overset{n\times n}{P}_t * \overset{n\times 1}{X}_{t-1} + \overset{m\times n}{H} \overset{n\times 1}{R}_t + \overset{m\times n}{H} * \overset{n\times m}{\tilde{Q}}_t * \overset{n_\varepsilon\times 1}{\varepsilon}_t$$

- implying that, if we know the path of X_t, the shocks can be obtained as

  $$\varepsilon_t = \left[H\tilde{Q}_t\right]^{-1} [Y_t - HP_tX_{t-1} - HR_t]$$

- Hence, assuming a sequence of contiguous shocks from time t=1 onward we can use the following
  - **STEP 1:** given a guess for the regimes $\left\{N_t^0\right\}$, modify *mkdatap_anticipated.m* to back out the shocks
    - Start from the first period with the economy in SS
      $$\varepsilon_1 = [HQ_1]^{-1} [Y_1 - HP_1X_0 - HR_1]$$
    - Then compute X_1 using
      $$X_1 = P_1X_0 + R_1 + Q_1\varepsilon_1$$
    - Then for $t = 2 : T_{shock}$ obtain
      $$\varepsilon_t = [HQ_t]^{-1} [Y_t - HP_tX_{t-1} - HR_t]$$
      $$X_t = P_tX_{t-1} + R_t + Q_t\varepsilon_t$$
  - **STEP 2:** given the shocks obtain the implied new set of regime $\left\{N_t^1\right\}$
    - For this you can simply modify solve**_oneconstraint.m** and adjust the output so that it spits out the regimes path